Academic Report (2020-21)

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Research Summary:

An algebraic structure $(E, +, \circ)$ is said to be a left skew brace if $(E, +)$ and $(E, \circ)$ are groups and, for all $a, b, c \in E$, the following compatibility condition holds:

$$a \circ (b + c) = (a \circ b) - a + (a \circ c).$$

A right skew brace can be defined analogously. A left skew brace is called a left brace if $(E, +)$ is an abelian group. This concept was introduced by Guarnieri and Vendramin in connection with set theoretic solutions of the quantum Yang-Baxter equation. Cohomology theory for braces acting trivially on abelian groups (viewed as trivial braces) was recently introduced and investigated by Lebed and Vendramin. Bachiler suggested some ideas about non-trivial actions. Let H be a left skew brace and I an abelian groups viewed as a trivial left brace. Generalising the ideas developed so far, we investigate second cohomology group of H acting on I non-trivially. A bijective correspondence between the following is established: (i) second cohomology group of a left skew brace H with coefficients in a trivial brace I; and (ii) class of extensions $0 \to I \to E \to H \to 0$ of H by I with the given action of H on I. We develop general cohomology theory in case the extension $0 \to (I, +) \to (E, +) \to (H, +) \to 0$, viewed as an extension of $(H, +)$ by $(I, +)$, splits. Further, we construct a fundamental exact sequence connecting the second cohomology group of a skew brace with the automorphism group of extensions of the skew brace. Such a sequence for groups was constructed by Wells in 1971.

A long standing well known problem in group theory is to classify finite groups in which every element of the commutator subgroup is a commutator. It has already been proved that in a finite simple group every element of the commutator subgroup is a commutator. At the extreme end, for finite p-groups the situation is quite different. It has been recently proved that if the commutator subgroup of a finite p-group, $p \geq 5$, is minimally generated by at most 3 elements, then every element of the commutator subgroup is a commutator. It is also known that the same happens for all p-groups upto order p^5. We recently classified finite p-groups G, p odd, such that the commutator subgroup $\gamma_2(G)$ of G is of order p^4 and exponent p, and $\gamma_2(G)$ contains a non-commutator. Continuing in this direction, we investigate finite p-groups of order p^7, and present a fine characterisation of those groups G of order p^7 whose $\gamma_2(G)$ contains elements which are not commutators in G. Our work, along with the existing theory, suggests a general phenomenon that might take place in all finite p-groups.

Publications:

1. Rahul Kaushik and Manoj K. Yadav, *Commutators and commutator subgroups in finite p-groups,* Journal of Algebra **568**, 314-348, (2021).

2. Silvio Dolfi, Anupam Singh and Manoj K. Yadav, *p-power conjugacy classes in $U(n, q)$ and $T(n, q)$,* Journal of Algebra and Its Applications **20**, 2150121 (13 pages), (2021).

Preprints:

1. Nishant and Manoj K. Yadav, *Cohomology, extensions and automorphisms of skew braces,* https://arxiv.org/abs/2102.12235.

2. Rahul Kaushik and Manoj K. Yadav, *Commutators in groups of order p^7,* http://arxiv.org/abs/2106.07205.

Conference/Workshops Attended:

1. *35th Annual Conference of RMS*, India, December, 2020.

2. *Group Theory Sangam*, India, January - May (weekly), 2021.

Invited Lectures/Seminars:

1. *The Schur multiplier of central product of groups*, GOThiC, Online version of Ischia Group Theory Conference, University of Salerno, Ischia, Virtual, June, 2020.

2. *Surjectivity of commutator map for finite p-groups*, Algebra Seminar, Haifa University (Israel), Haifa, Virtual, November, 2020.

3. *Commutators in finite groups*, 80th Refresher Course, Punjabi University, Patiala, Virtual, November, 2020.

4. *Surjectivity of commutator map for finite p-groups*, Departmental Seminar, IISER TVM, Virtual, December, 2020.

5. *Group theory is the living room*, Refresher Course (Mathematics and Statistics) , Guru Jambheshwar University of Science and Technology, Hisar, Virtual, January, 2021.

6. *Left braces*, In house Symposium, IISER Bhopal, Virtual, March, 2021.

Other Activities:

1. Organised a semester long online weekly seminar series "Group Theory Sangam", January - May, 2021 (jointly with Anupam Singh, IISER Pune).

2. Served as an editorial board member of Proc. Math. Soc., IASc.

3. Refereed research papers for national and international journals.

4. Instructed Arpan Kanrar for him first semester graduate Project Course at HRI.

5. Served HRI on several committees.