# Academic Report (2020-21)



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## Research Summary:

During this academic year, we studied the linear independence of some real numbers. More precisely, we proved the following result. *Let $\beta$ be a P-V number and $m \geq 2$ be any integer. Let $a_1, \ldots, a_m$ be distinct positive integers. Then the numbers*

$$1, \sum_{n=1}^{\infty} \frac{1}{\beta^{a_1 n^2}}, \sum_{n=1}^{\infty} \frac{1}{\beta^{a_2 n^2}}, \ldots, \sum_{n=1}^{\infty} \frac{1}{\beta^{a_m n^2}}$$

*are $\mathbb{Q}(\beta)$-linearly independent.* Note that by a result of Elsner, Luca and Tachiya, it follows each one of the series in the above result is transcendental. However, when $m \geq 3$, these numbers are algebraically dependent over $\mathbb{Q}$. As a consequence of the above result, we obtain the following. *Let $\beta$ and $a_i$'s be as in the above Theorem. By letting $\tau_n = \frac{i \log \beta}{\pi} + 2n$ for each natural number $n \geq 1$, we have the $m$-Jacobi theta values*

$$\theta_3(a_1 \tau_n), \quad \theta_3(a_2 \tau_n), \ldots, \theta_3(a_m \tau_n)$$

*are $\mathbb{Q}(\beta)$-linearly independent with* 1. Also, another consequence is: *Let $\beta > 1$, $a_i$'s and $\tau_n$ be as in the above Corollary. Then for every integer $n \geq 1$, the $m$-theta values*

$$\theta_4(a_1 \tau_n), \quad \theta_4(a_2 \tau), \ldots, \theta_4(a_m \tau_n)$$

*are $\mathbb{Q}(\beta)$-linearly independent with* 1. Here $\theta_3(\tau)$ and $\theta_4(\tau)$ are Jacobi-Theta functions on complex upper half plane. Other than this result, we also studied the simultaneous approximation of algebraic numbers and as a consequence, we proved a transcendental criterion.

Most of this academic session is devoted to a well-thought book on Algebraic number theory from scratch to advanced topics. The first draft of the book was completed in this academic session.

## Publications:

1. R. Thangadurai and N. Saradha, *Pillars of Transcendental Number Theory,* Springer, Singapore, (2020)

## Preprints:

1. R. Thangadurai, Debasish Karmakar and Veekesh Kumar, *Linear Independence of special values of Jacobi-Theta constants*, 2021.

2. R. Thangadurai and Veekesh Kumar, *On Simultaneous Approximation of Algebraic numbers,* arXiv:2001.00386v2.

3. R. Thangadurai and S. A. Katre, *Algebraic Number Theory* (In preparation).

4. R. Thangadurai and S. D. Adhikari, *Zero-sum Theorems in Additive Combinatorics: An introduction* (In preparation).

## Conference/Workshops Attended:

1. *Teacher Enrichment Workshop*, India, February, 2021.

2. *Balu 70 Number Theory Fest*, India, March, 2021.

## Invited Lectures/Seminars:

1. *Decimal Expansion, Gauss Conjecture and Borel Conjecture*, National Webinar, Dwaraka Doss Goverdhan Doss Vaishnav College, Chennai, July, 2020.

2. *Quadratic Reciprocity Law*, Refresher Course, Vaishnav College, Chennai, November, 2020.

3. *On simultaneous approximation of algebraic numbers*, Algebra/Number Theory Seminar, Indian Institute of Science, Bangalore, December, 2020.

4. *3 by 3 magic squares and A.P. in rational points on elliptic curves*, Ramanujan Birthday Seminar, Pi Institute, Chennai, December, 2020.

5. *Introduction to Complex Analysis*, Teacher Enrichment Workshop, Kongu Engineering College, Perundurai, February, 2021

6. *Linear Independence of Special Values of Jacobi Theta Constants*, Balu 70 Number Theory Fest, Institute of Mathematical Sciences, Chennai, March, 2021.

## Academic recognition/Awards:

- Managing Editor, Hardy Ramanujan Journal, 2020-21.

## Other Activities:

1. *Given reading course on 'Rational Approximations and Introduction to Transcendental Number Theory*, Mr. Kevin Alex, Centre for Basic Sciences, Mumbai, January - May, 2021.

2. *Reading course on 'Algebraic Number Theory'*, Mr. Arpan Das, HRI, January - July, 2021.

3. *Member of HRI Covid Monitoring Committee*, Since March, 2020

4. *Done NAAC exercise for HBNI* during 2020-21.

5. *Nodal officer for PSA exercise*, August-November, 2020.