Academic Report (2022-23)

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Research Summary:

In 1919, G. Polya proved that the given algebraic number α is an algebraic integer if and only if $Tr(\alpha^n) \in \mathbb{Z}$. Then Lame posed a question that if a non-zero algebraic number α satisfies $Tr(\alpha^m) \in \mathbb{Z}\backslash\{0\}$ for infinitely many natural numbers m, is it true that α is an algebraic integer?. This question was solved by Corvaja and Zannier in 2004 and generalized by Philippon and Rath in 2021. We looked into the following problem. *Let $\alpha_1, \ldots, \alpha_n, \lambda_1, \ldots, \lambda_n$ be non-zero algebraic numbers. Does $Tr(\lambda_1\alpha_1^m + \cdots + \lambda_n\alpha_n^m) \in \mathbb{Z}\backslash\{0\}$ for infinitely many natural number m imply that each α_i is an algebraic integer?* One can obtain a counter-example to this question. Thus, we needed to understand that if the linear recurrence sequence $\lambda_1\alpha_1^m + \cdots + \lambda_n\alpha_n^m$ are algebraic integers for infinitely many natural numbers m, then what kind of exhaustive cases arises. Precisely we found what cases arises and we obtain many results in this directions. We also could prove a finite version in the spirit of De Smit when $n = 2$. This is a joint work with A. Bharatwaj, A. Pal and V. Kumar.

In another work, along with A. Tripathi, we are studying the $\mathbb{Q}$-linearly independence of the following values of the infinite series. More generally, we are exploring the $\mathbb{Q}$-linearly independence of

$$1, \sum_{m=1}^{\infty} \frac{1}{[1,2,\ldots,mq+1]}, \sum_{m=1}^{\infty} \frac{1}{[1,2,\ldots,mq+2]}, \ldots, \sum_{m=1}^{\infty} \frac{1}{[1,2,\ldots,mq]}$$

where q is a prime number and $[1, 2, \ldots, n]$ denotes the least common multiple of the numbers $1, 2, \ldots, n$?. We have solved this question when $q = 2$ and $q = 3$. Moreover, we have also proved that the value of the series

$$\sum_{m=1}^{\infty} \frac{1}{[1,2,\ldots,mn+r]}$$

is an irrational number for any natural number n and $r \in \{0, 1, 2, \ldots, n-1\}$. We are looking at the possibility of proving their transcendence nature.

Publications:

1. V. Kumar and R. Thangadurai, *On simultaneous approximation of algebraic numbers*, Mathematika **68** (2022), no. 4, 1153-1175.

2. M. Makeshwari, V. P. Ramesh and R. Thangadurai, *A necessary and sufficient condition for* 2 *to be a primitive root of* $2p+1$, Math. Student **89** (2020), no. 3-4, 171-176.

Preprints:

1. A. Bharadwaj, A. Pal, V. Kumar and R. Thangadurai, *Sufficient conditions for classifying algebraic integers*, Preprint, (2022).

2. A. Tripathi and R. Thangadurai, *On the irrationality of certain infinite series*, In preparation, (2023).

Other Activities:

1. Guided a master thesis student, Ms. Sanskruti Karnawat, CBS Mumbai on Elliptic Curves during August 2021 - July 2022.
2. Guided a master thesis student, Mr. Vaibhav Gupta, NIT Surat on "Hilbert Irreducibility Theorem" during December 2022 - April 2023.
3. Co-guide of a Ph.D student, Mr. Debashish Karmakar, HRI during August 2015-December 2022.
4. Organized a one-day meet on "Algebra and Number Theory" at HRI, Prayagraj on 10th September 2022.
5. Organized NCM Workshop on 'Elliptic Curves, Elliptic Functions and Transcendence" at HRI, Prayagraj during 24, November - 03, December 2022.
6. Served as a Nodal Officer at HRI for HBNI during this academic year.